Solutions
Math 220
HW # 10
November 29, 2018

Exercise 1. *Let* $A = \{2, 3, 4\}$ *and* $B = \{6, 8, 10\}$ *and define a binary relation* R *from* A *to* B *by* xRy *if and only if* $x|y$.

(a) *Is* $4R6$*? Is* $4R8$*? Is* $(3, 8) \in R$*? Is* $(2, 10) \in R$*?*

(b) *Write* R *as a set of ordered pairs.*

Solution.

(a) (i) No, (ii) Yes, (iii) No, (iv) Yes

(b) $R = \{(2, 6), (2, 8), (2, 10), (3, 6), (4, 8)\}$.

□

Exercise 2. *Let* $n \in \mathbb{N}$. *Define a relation* $\sim$ *on* $\mathbb{Z}$ *by* $a \sim b$ *if and only if* $a \equiv b \mod n$. *Show that* $\sim$ *is reflexive, symmetric, and transitive.*

Proof.

(1) $\sim$ is reflexive:
Let $a \in \mathbb{Z}$. Then since $a - a = 0$, we know that $n|(a - a)$ because $n|0$, so $a \sim a$.

(2) $\sim$ is symmetric:
Suppose $a \sim b$. Then $a \equiv b \mod n$, or $n|(a - b)$. This means $a - b = nk$ for some $k \in \mathbb{Z}$. It then follows that

$$b - a = -(a - b) = -nk = n(-k)$$

which shows that $n|(b - a)$ hence $b \equiv a \mod n$. Therefore $b \sim a$.

(3) $\sim$ is transitive:
Suppose that $a \sim b$ and $b \sim c$. Then $a - b = nk$ and $b - c = nl$ for some $k, l \in \mathbb{Z}$. Adding these two statements gives

$$(a - b) + (b - c) = nk + nl \iff a - c = n(k + l).$$

Thus $a \equiv c \mod n$ and so $a \sim c$.

□